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A new method for computing the centre of mass of a bicycle and rider

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Abstract
For developing riding skills on mountain bikes, it is important to know how the centre of mass of a bicycle and its rider changes with ground inclination or with rider position. We show here a new method for finding the location of this point by measuring the normal forces acting on the wheels in two positions with a digital weight indicator. This method can be easily applied in the classroom and can be used as a real-life example for computing the centre of mass of a complex body.

Introduction

The centre of mass of a body is the average position of all of its particles weighted by their masses (Young and Freedman 2000). For a homogeneous body, the centre of mass is identical to its geometrical centre. For a body that is built up from several masses, $m_1, m_2, \ldots m_n$, located at positions $(x_1, y_1, z_1), (x_2, y_2, z_2), \ldots (x_n, y_n, z_n)$, the centre of mass of the system is located at $(x_{\rm cm}, y_{\rm cm}, z_{\rm cm})$ and is given by

$$x_{\rm cm} = \frac{1}{M}\sum_{i=1}^{n} m_i x_i \qquad y_{\rm cm} = \frac{1}{M}\sum_{i=1}^{n} m_i y_i, \qquad z_{\rm cm} = \frac{1}{M}\sum_{i=1}^{n} m_i z_i, \tag{1}$$

where $M = \sum_{i=1}^{n} m_i$ is the total mass. It is impractical to use this method for more complicated systems, such as a bicycle and rider, which are built from many parts with different densities. In the literature, many use the 'the typical bicycle and rider' of Whitt and Wilson, in which 90.7 kg is used for the mass of the bike and rider and 1.067 m for the wheel base (Wilson 2004). But they do not describe how they obtained these numbers. For Wilson's typical bicycle and rider $x_{\rm cm} = 0.432$ m and $y_{\rm cm} = 1.143$ m measured from the rear wheel ' base. Here, we suggest a new method to find the centre of mass of the bicycle–rider system, which can be carried out with a digital weight indicator. The location of the centre of mass of the rider and the bicycle is further dependent on the frame geometry and it will also depend on the rider's saddle height and his riding position. It is known that mountain bike riders lower their saddle during downhill technical sections and that they have to shift their weight backwards in order to control the movement of their bike (Trombley 2005).

Method

The basic idea here is to measure the normal forces acting on the two wheels for two different positions, horizontal and inclined, where the

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Figure 1. Measuring the normal force acting on the rear wheel of a road bike with a digital weight indicator. To help him stay balanced without support, for a brief time the rider has to use the wall and a person standing on the other side of the bike.

bicycle and the rider creates an angle α with the plane and to apply Newton's law for rotational equilibrium. We measured the normal forces acting on the wheels by using a digital weight indicator, as shown in figure 1. To stay balanced on the bike the rider can push against the wall and lean on the shoulders of another person. It may take some practice to stay freely balanced on the bike, especially for the measurement of the inclined case.

Horizontal case

When the bicycle and the rider are horizontal to the ground, the forces acting on them are illustrated in figure 2. We place the origin at the centre of the rear wheel; L_0 is the distance between the rear and front wheel contact points; and x_0 is the horizontal displacement from the origin to the centre of mass (CM).

For the system to be in equilibrium, the sum of all the torques acting on the bike around any point must be zero. A torque may be written as

$$\vec{\tau} = \vec{r} \times \vec{F} = rF \sin\theta \hat{n}, \tag{2}$$

where $\vec{r}$ is the directed distance from the centre of rotation to the point where the force $\vec{F}$ is applied, and θ is the angle between $\vec{r}$ and $\vec{F}$. The unit vector $\hat{n}$ points out of the paper (the positive direction) when the torque acts to rotate the rigid body in the counterclockwise direction.

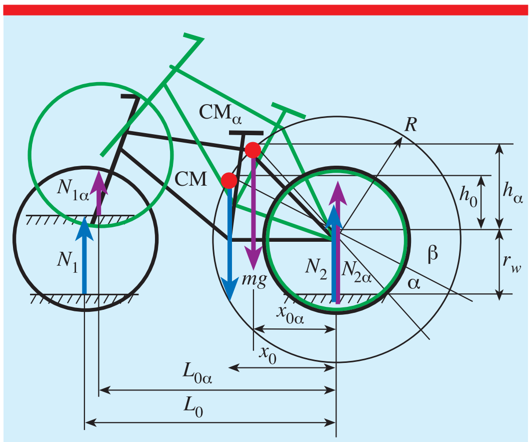

Figure 2. Schematic diagram of forces acting on the bike in the horizontal and inclined positions (for clarity the rider is not shown). N_1 and $N_{1\alpha}$ are the normal forces acting on the front wheel for the horizontal and inclined states, respectively, and N_2, $N_{2\alpha}$ are those for the rear wheel. CM and CM$_\alpha$ are the locations of the centre of mass for the horizontal and the inclined positions, respectively.

Figure 3. Inclined bike. In order to easily measure $L_{0\alpha}$ we hang small weights from the rear and front hubs, which serve as plumbs (see the arrows). Note that the system configuration is only an approximation for a real slope as the bicycle rests on horizontal slabs.

Equating the opposing torques operating around the origin gives

$$N_1 L_0 = mgx_0, \tag{3}$$

which can be written as $x_0 = (N_1/mg)L_0$ and where g is the acceleration due to gravity.

As L_0 and N_1 can be measured and mg is known, x_0 can be calculated. To determine the height of the centre of mass, we have to measure the normal forces on a bike inclined at angle α.

Inclined case

For this case, we lift the front of the bike around the fixed rear wheel axle and put a few wood slabs under the front wheel, as shown in figure 3. The new centre of mass, CM_α, is still located on the circle of radius R, whose centre is at the origin of our chosen coordinate system.

The geometry of the problem is illustrated in figure 4. The centre of mass, CM_0, which lies along a circle with radius R centred around the origin, initially formed an angle β when the bike was horizontal to the ground. In the inclined case, the CM_α moves an additional α degrees clockwise along this circle. The horizontal distance between the two wheels, which can be measured, is $L_{0\alpha}$ (note that $L_{0\alpha} < L_0$). $x_{0\alpha}$ is the horizontal distance of CM_α from the origin. Note that the

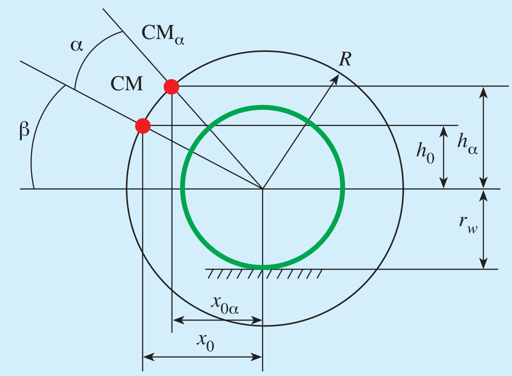

Figure 4. The geometry of the change in position of the centre of mass from inclining the bike is seen in this enlargement of the rear wheel. The line connecting the centre of mass of the horizontal bike and the rear wheel makes an angle β with the x-axis, whereas that of the inclined bike (CM_α) makes an angle α + β. h_0 and h_α are the y-coordinates of CM_0 and CM_α, respectively, and r_w is the wheel radius.

Table 1. The normal forces acting on the wheels of a road and mountain bike and a rider (his mass was 76.1 kg) measured by a digital weight indicator. The forces are given in Newtons. The relative error of each measurement compared with the total weight of the rider and bike is indicated in percentage. The errors are due the difficulties in measuring the weight because it is hard to stay balanced on the bike without any support.

	N_1 (N) front wheel	N_2 (N) rear wheel	$N_1 + N_2$ (N)	$N_{1\alpha}$ (N) front wheel	$N_{2\alpha}$ (N) rear wheel	$N_{1\alpha}+N_{2\alpha}$ (N)
Road bicycle (847 N)	368	521	889 (5%)	177	700	877 (3.5%)
Mountain bike (879 N)	350	582	932 (6%)	168	746	914 (4%)

actual distances of CM or CM_α from the origin are the same as that defined as R (see figure 4). $N_{1\alpha}$ and $N_{2\alpha}$ are the normal forces acting on the front and rear wheels respectively. Parallel to equation (2), we can write

$$x_{0\alpha} = \frac{N_{1\alpha}}{mg} L_{0\alpha}. \tag{4}$$

As $L_{0\alpha}$ (see figure 4) and $N_{1\alpha}$ can be measured and mg is known, x_0 can be calculated.

Using simple trigonometric relations, we can express R by the following two equations:

$$\begin{aligned} R &= x_0/\cos\beta \\ R &= x_{0\alpha}/\cos(\alpha+\beta). \end{aligned} \tag{5}$$

The angle β can then be found by solving the equation:

$$\frac{x_{0\alpha}}{\cos(\alpha+\beta)} = \frac{x_0}{\cos\beta}. \tag{6}$$

Using the formula for a cosine of a sum of two angles:

$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta, \tag{7}$$

we get

$$\cos\beta(x_0\cos\alpha - x_{0\alpha}) = x_0\sin\alpha\sin\beta, \tag{8}$$

which gives an expression for $\tan\beta$:

$$\tan\beta = \frac{x_0\cos\alpha - x_{0\alpha}}{x_0\sin\alpha}. \tag{9}$$

The height h_0 is then

$$h_0 = x_0\tan\beta. \tag{10}$$

The height of the centre of mass above the ground is $H = h_0 + r_w$, where r_w is the rear wheel radius. h_0 can be explicitly written in terms of the normal forces N_1 and N_2. Using equations (3) and (4) for $x_0 = (N_1/mg)L_0$ and $x_{0\alpha} = (N_{1\alpha}/mg)L_{0\alpha}$ in equation (9) gives:

$$\tan\beta = \cot\alpha - \frac{N_{1\alpha}L_{0\alpha}}{N_1 L_0 \sin\alpha}. \tag{11}$$

Using the fact that $L_{0\alpha} = L_0\cos\alpha$ and equation (10), it can be easily shown that

$$h_0 = \frac{L_0\cot\alpha(N_1 - N_{1\alpha})}{mg}. \tag{12}$$

Note that both equations (10) and (12) use two measurements of normal forces acting on the front wheel, one horizontal and one in the inclined case, and geometric measurement of $L_{0\alpha}$. The measurements of the normal forces are more difficult to perform and insert inaccuracy in the results.

Results

We have done measurements for two kinds of bikes: a road bike (specialized Allez, $m = 10.3$ kg) and a mountain bike (specialized HR. XC, $m = 13.6$ kg) with the same rider ($m = 76.1$ kg). Note that the results listed below include the rider mass. The results are summarized in table 1. For the road bicycle $L_0 = 0.98$ m, $L_{0\alpha} = 0.9$ m, which gives $\cos\alpha = L_{0\alpha}/L_0 = 0.939$ so $\alpha = 20.11°$ and $x_0 = 41.45$ cm, $h_0 = 58.78$ cm. Thus, the height of the centre of mass is $H = 58.78 + 33.5 = 92.28$ cm.

A similar calculation for the mountain bike gives $x_0 = 32.5$ cm, $\cos\alpha = L_{0\alpha}/L_0 = 0.958$, $\alpha = 16.67°$, $h_0 = 39.01$ cm and $H = 39.01 + 33.5 = 72.51$ cm.

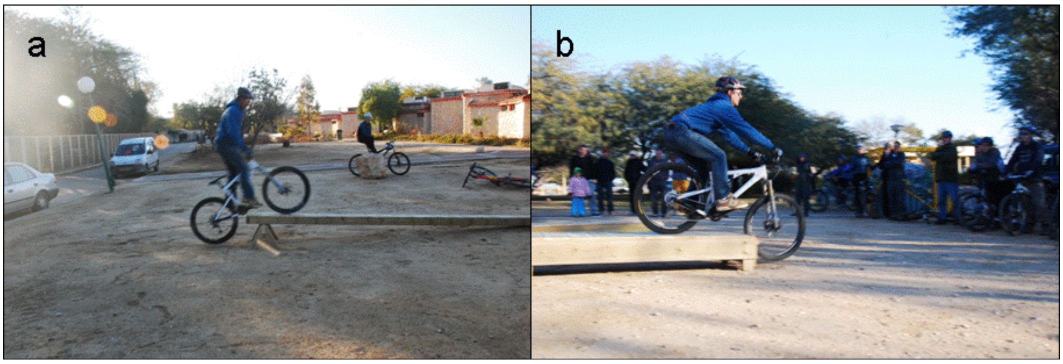

Figure 5. When a rider is going up a step he moves his CM forward (a). The situation is opposite when going down a step (b). These pictures shows a configuration similar to the one we used for measuring the CM.

Conclusions

We have developed a simple method for locating the centre of mass for a rider and his bicycle. The results show that for a bicycle on a flat plane 60% of the total force falls on the rear wheel and 40% on the front wheel. However, on an inclined plane with a slope of 17°, 80% of the total force falls on the rear wheel whereas only 20% falls on the front wheel. Thus, when going uphill with a mountain bike, the front wheel can lose traction and the rider has to pull his upper body forward to increase the normal force acting on the front wheel (Wehrbein 2004). The situation is changed when going downhill and the rider has to move his weight back to increase the rear wheel traction as shown in figure 5. Our results show that the height of the centre of mass of the mountain bike is about 20 cm below that of the road bike. For mountain bike riding, this height is very important and influences the stability of the bike on different terrains (Lopes and McCormack 2005). Therefore, the rider will often lower his seat before going downhill or along rocky sections to lower the height of bike and rider and their combined centre of mass. In the lab, students can explore how CM on the same bike depends on the mass of the rider by carrying out measurements for light and heavy riders.

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References

Lopes B and McCormack L 2005 *Mastering Mountain Bike Skills* (China: Human Kinetics) p 20
Trombley A 2005 *Serious Mountain Biking* (Champaign, IL: Human Kinetics) p 58
Wehrbein W M 2004 Wheelies and headers, or how to keep both bicycle wheels on the ground *Phys. Teach.* **42** 27–30
Wilson D G 2004 *Bicycling Science* 3rd edn (Cambridge, MA: MIT Press) p 244
Young H D and Freedman R A 2000 *University Physics* 10th edn (San Francisco: Addison-Wesley) p 330

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