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A simple model of aeolian megaripples

H. Yizhaq*

BIDR, Ben Gurion University, Sede Boker Campus 84990, Israel

Abstract

We discuss a simple mathematical model for the formation of aeolian megaripples. The main idea is that the sediment consists of a mixture of grains with two different sizes and that the wind is not strong enough to cause the coarse fraction to saltate. Linear stability analysis indicates the presence of two maxima in the growth rate of the unstable modes. The gravest mode corresponds to megaripples and the other to "standard" aeolian ripples.

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Keywords: Sand ripples; Megaripples; Saltation; Reptation

1. Introduction

Aeolian sand ripples are a common feature in sandy deserts and on beaches. Standard aeolian ripples have wavelength of a few tens of cm, and amplitude of a few mm [1]. Sometimes, much larger ripples are observed. These latter have been termed in different ways, such as ridges [1], granule ripples [2], megaripples [3] or gravel ripples [4]. Aeolian megaripples have been observed in several locations on Earth (see Fig. 1) [1,2,5–9] and on Mars [8,10,11]. Aeolian megaripples are composed by a mixture of coarse and fine non-cohesive material; a bimodal distribution of particle sizes is thought to be necessary for large ripple-like bedforms to develop. The coarse grains account for 50–80% of the surface material at the crestal area and for no more than 10–20% in the troughs [2]. Megaripples are characterized by an asymmetric profile, with wavelength up to 20 m and amplitude of tens of cm [10]. There is a correlation between the ripple height and the ripple wavelength, with a ripple index of approximately 15, and between the megaripple wavelength and the maximum particle size

* Corresponding author. Fax: +972-8-6596921.
E-mail address: yiyeh@bgumail.bgu.ac.il (H. Yizhaq).

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Fig. 1. Sand ripples at Great Sand Dunes National Monument in Colorado (photograph by Bob Bauer). The foreground (lower part of the picture) shows sand ripples which exhibits a typical fingerprint-like pattern. The background (upper part of the picture) shows granule ripples with wavelength up to 2 m and height up to 9 cm [8].

(see Figs. 3 and 4 in Ref. [8]). Field observations indicate that often megaripples have normal ripples superposed on both their windward and lee slopes [10].

In this work we discuss a simple mathematical model for the formation of aeolian megaripples. Although many models of the dynamics of standard aeolian ripples are available (see e.g. Ref. [12]), the mathematical description of megaripple evolution has received little or no attention. Here, we build on the integro-differential aeolian ripple model proposed by Anderson [13] and recently extended by Yizhaq et al. [14], and include spatial variations of the saltation flux.

2. Saltation and reptation

Aeolian bedforms are generated by the action of wind on a surface composed of non-cohesive material such as dry sand or dry snow [1]. When a strong enough wind starts to blow on a dry sand surface, a first generation of grains is lifted and accelerated by the wind. The grains do not stay suspended as gravity pulls them down to the ground. Upon their impact with the surface, the grains impart their energy and momentum to the surface, and eject other grains. If the wind is strong enough, the newly ejected grains are again accelerated, fall down, and eject still other grains. In this way, a cascade process can be established. In the mature stage, the wind has the only role of accelerating the grains during their flight, and the dynamics is determined by the collision process. This grain–grain (or grain–surface) collision mechanism is typical of aeolian environments. Here, in fact, the large density difference between air and sand is thought to confine the direct effect of the wind shear stress to the early development of the process. A different situation is encountered for ripples under water, where the density difference between sand and water does not allow for the generation of an efficient bombardment process and the direct action of the bottom shear stress due to the water flow continues to be important also at later stages. Ripples and megaripples are almost one-dimensional bedforms, with long parallel crests perpendicular to the wind direction.

In the model adopted here, we assume the wind to have constant direction and focus on the behavior in the along-wind direction, which we call x (see Ref. [14] for a study of two-dimensional ripples). In the following, we shall formulate a mathematical description of the spatio-temporal evolution of the sand surface, $\zeta(x,t)$, in response to the action of wind and to the processes of saltation and reptation. The starting point is the Exner equation, a continuity equation for sand:

$$\zeta_t = -\frac{1}{\rho_p(1-\lambda_p)}\,\partial_x Q\ , \tag{1}$$

where $\zeta(x,t)$ is the local height of the sand surface at point x and time t, ρ_p is the density of a sand grain, λ_p is the porosity of the bed (typically taken as 0.35). Q is the horizontal flux of sand grains mobilized by the wind action. This equation shows that erosion (deposition) occurs in regions where the sediment flux is diverging (converging), and there is no change in the surface height where the transport rate is constant. Experimental results [13] indicate that the bombardment process generates two populations of moving grains: grains that are ejected with large energy and are accelerated by the wind form the population of "saltating" grains. The second population consists of grains that are ejected with low energy, and stay close to the sand surface, forming the so-called "reptating" population. The total sand flux is then the sum of the saltation (s) and reptation (r) fluxes, $Q = Q_s + Q_r$. Interestingly, the exchange flux between the two populations is thought to be (or hoped to be) small [13]. In the Exner equation, it is only the spatial variability of the sand flux that determines the evolution of the surface. For standard sand ripples, it is assumed that the ballistic, wind-buffeted trajectory of a saltating particle leaves that grain with no memory of its starting conditions. As a result, all grains are assumed to descend at a fixed angle and with a fixed speed, and the saltation flux becomes constant. In this case, only the spatial variability of the reptation flux enters the Exner equation [13]. This hypothesis is reasonable when the bed undulations have wavelength much smaller than the typical length of a saltation jump. For standard sand ripples, typical wavelengths are about 10–20 cm and the typical saltation length is about one meter, and the neglect of the variability of the saltation flux is justified. The assumption on the constancy of the saltation flux is not tenable for bedforms with larger wavelength, such as dunes and megaripples. In this case, both the saltation and the reptation fluxes affect the evolution of the sand surface.

In the following, we model the dynamics of a sediment surface which consists of a mixture of sand grains with two different sizes, coarse (c) and fine (f). For such a sand mixture, the saltation flux Q_s can be written as $Q_s = Q_{sf} + Q_{sc}$ where Q_{sf} (Q_{sc}) is the saltation flux of fine (coarse) particles. Similarly, the reptation flux Q_r can be written as $Q_r = Q_{rc} + Q_{rf}$. The total flux becomes $Q = Q_{sf} + Q_{sc} + Q_{rc} + Q_{rf}$. Consistent with observations, we introduce two basic simplifications in the model. First, we assume that the wind is not strong enough to drive coarse particles into saltation, i.e. $Q_{sc}=0$. Second, we neglect the reptation flux of fine grains, $Q_{rf}=0$, as it provides a small contribution to the total flux on megaripples due to the surface armoring effect of coarse particles [6,7,17]. We also assume Bagnold's necessary conditions for megaripples growth [1, p. 156] and consider a continued supply of fine grains to provide reptation of coarse grains.

3. Model formulation

The model is based on the Exner equation (1), where only Q_{rc} and Q_{sf} are retained in the flux term. Following Refs. [12,14], we write the reptation flux as

$$Q_{rc} = m_c n_p (1 - \mu_r \zeta_x) \int_0^\infty \mathrm{d}\alpha\, p_{rc}(\alpha) \int_{x-\alpha}^{x} N_{im}(x')\, \mathrm{d}x' \ , \tag{2}$$

where $N_{im}(x,t)$ is the number density of impacting saltating grains at the position x and time t, to be further discussed below, $p_{rc}(\alpha)$ is the so-called "splash function" that describes the distribution of the reptation lengths, m_c is the mass of a coarse grain and n_p is the average number of reptating grains ejected by the impact of one (fine) saltating grain. The parameter μ_r heuristically includes both the bed slope modification of the ballistic trajectory and the Hardisty and Whitehouse correction ("impact-induced gravity flow") as discussed in Ref. [14]; the magnitude of μ_r must be determined empirically. Similarly, we write the saltation flux of the fine particles as

$$Q_{sf} = m_f (1 - \mu_s \zeta_x) \int_{-\infty}^\infty \mathrm{d}\beta\, p_{sf}(\beta) \int_{x-\beta}^{x} N_{im}(x')\, \mathrm{d}x' \ , \tag{3}$$

where $p_{sf}(\beta)$ is the distribution of the saltation lengths, m_f is the mass of a fine grain and μ_s is a phenomenological parameter which takes into account the bed slope dependence of the saltation flux. We assume that also the saltation flux decreases on the windward slope of the (mega) ripple and increases on the lee slope. Consistent with observations, we have assumed that on average only one saltating grain is ejected for each impacting grain. At this point we have to estimate $N_{im}(x,t)$. First of all, the density of impacting grains changes because of local variations in the bed slope. Based on geometrical considerations, we obtain

$$N_{im}(x) = N_{im}^0 \left(1 + \frac{\tan\theta}{\tan\phi}\right) \cos\theta = N_{im}^0 \frac{1 + \zeta_x \cot\phi}{\sqrt{1+\zeta_x^2}} \ , \tag{4}$$

where N_{im}^0 is the number density of impacting grains on a flat surface and θ is the inclination of the bed (θ is positive at the windward slope). For standard aeolian ripples, the saltation flux on a flat surface is assumed to be uniform and thus N_{im}^0 is a constant. Here, however, we admit also the saltation flux to be variable. The variability of the saltation flux leads to space–time dependence of N_{im}^0, which now depends on the number of grains ejected at $x-\beta$ that jumped the distance β. At this point, the problem becomes quite complicated. In the following, we focus only on linearized dynamics, and for simplicity we assume N_{im}^0 to be constant. We should keep in mind, however, that this is an approximate treatment whose validity is purely heuristic.

We next study the behavior of the linearized version of the Exner equation (1), with $Q = Q_{sf} + Q_{rc}$ for the sand flux. To expedite comparison with experimental and observational data, we present the linear stability analysis results in dimensional form. We assume an infinitesimal sinusoidal perturbation on a flat granular bed, given by

$$\zeta(x,t) = \zeta_0 \exp(\mathrm{i}k(x - ct)) , \tag{5}$$

where ζ_0 is the amplitude of the perturbation. Following the calculation as done in Ref. [14], we get

$$c = \varepsilon Q_0(1 - \hat{p}_{sf}(k) - \mathrm{i}k\mu_s b \tan\phi) + n_p \delta Q_0(1 - \hat{p}_{rc}(k) - \mathrm{i}k\mu_r a \tan\phi) , \tag{6}$$

where $\hat{p}_{sf}(k)$ and $\hat{p}_{rc}(k)$ are the Fourier transforms of the probability distributions and c is the (complex) growth rate of infinitesimal perturbations. Here, $Q_0 = N_{im}^0 m_f \cot\phi/[\rho_p(1-\lambda_p)]$ and $\delta = m_c/m_p = (D_c/D_f)^3$ where D_c and D_f are the diameters of coarse and fine grains, respectively, assuming identical density and spherical shape. According to Bagnold [1, p. 154] a saltation grain can sustain a forward movement of a coarse grain with a diameter 3–7 times larger than its own diameter thus, $27 \leqslant \delta \leqslant 343$. The step lengths are chosen from an exponential distribution [18], $p_{rc} = \mathrm{e}^{-\alpha/a}/a$, where a is the mean reptation length defined as $a = \int_0^\infty \alpha p(\alpha)\,\mathrm{d}\alpha$ and $\int_0^\infty p(\alpha)\,\mathrm{d}\alpha = 1$; $p_{sf}(\beta) = A\mathrm{e}^{-(\beta-b)^2/(2\sigma^2)}$ is taken as a normal distribution [18], where $A = 1/(\sigma\sqrt{2\pi})$ and $\int_{-\infty}^{\infty} p_{sf}(\beta)\,\mathrm{d}\beta = 1$, b is the mean saltation hop length defined as $b = \int_{-\infty}^{\infty} \beta p_{sf}(\beta)\,\mathrm{d}\beta$ and σ is the standard deviation.

A further step concerns the fraction of fine to coarse sand in the sediment mixture. We model this effect by introducing a phenomenological parameter, ε, which is the ratio between the amount of coarse and fine grains on a flat surface. i.e., we assume the (fine grain) spatial variability of the saltation flux to be proportional to ε, mainly because the surface relief becomes higher as coarse grains accumulations occur in certain locations. In addition, the mean saltation path can be very large (up to several meters) when a small grain is bounced off a larger one [16]. Thus, $\varepsilon = 1$ means equally distribution of fine and coarse grains and $\varepsilon = 0$ stands for unimodal fine sand. Note that for $\varepsilon = 0$ the contribution of the fine saltation flux to the evolution of the local height of the sand surface vanishes and the model reduced to the case of normal sand ripples. The percent of the coarse grains of all the surface material or the weight percent [2] p can be expressed as $p = 100\varepsilon/(1+\varepsilon)$. Note that in this formulation there is no explicit distinction between the two kinds of grains, therefore our model cannot explain the segregation of grains inside the ripple. Linear instability occurs for $c_i > 0$ where c_i/Q_0 is given by

$$\frac{c_i}{Q_o} = k\left\{\frac{\delta n_p a^{-1}}{k^2 + a^{-2}} - \tan\phi(\delta n_p \mu_r a + \varepsilon\mu_s b)\right\} + \varepsilon \exp\left(-\tfrac{1}{2}\sigma^2 k^2\right) \sin bk . \tag{7}$$

Linear stability analysis indicates the presence of two maxima in the growth rate curves. The gravest mode corresponds to megaripples and the other to "standard" aeolian ripples. Fig. 2a shows a growth curve with two maxima for $\delta = 27$, the first maximum pertains to ripples with $\lambda_s = 2\pi/k_s \approx 157$ cm and the second to ripples with a much shorter wavelength $\lambda_r = 2\pi/k_r \approx 2.09$ cm. Fig. 2b shows the growth rate curves for different values of ε and indicates the existence of a minimum percent of the coarse

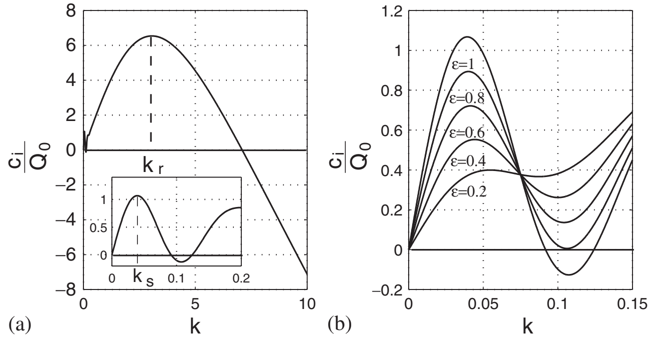

Fig. 2. (a) Dispersion curve showing two maxima. The first one pertains to megaripple and the second pertains to normal sand ripples. The inset shows in more details the curve for $0 \leqslant k \leqslant 0.15$; (b) Growth rate curves for different values of ε. For $\varepsilon \sim 0.8$ ($p_{min} = 44.4$) the gravest mode disappears. The existence of such a threshold agrees roughly with Sharp's observation [2, p. 635] that a concentration of coarse grains giving a surface coverage of at least 50% in the crestal area is needed for granule ripples formation. Parameters: $\varepsilon = 1$ $\phi = 10^0$, $b = 40$ cm, $\sigma = 10$ cm, $\delta = 27$, $a = 0.2$ cm, $\mu_s = 0.2$, $\mu_{rc} = 0.4$, $n_p = 1$.

fraction at the surface material that is necessary for megaripples formation. The model predicts that the megaripple wavelength is about several times the mean saltation hop length. The instability is due to the geometrical fact that the windward face is exposed to more bombardment particles than the lee slope. The idea that the megaripple wavelength is related to the mean saltation length was suggested by Ellwood et al. [16] and it goes back to the old idea of Bagnold [1] about the "characteristic length".

4. Conclusions

The simplified model presented here takes into account both saltation and reptation flux in the formation of sand ripples. Following Bagnold [1] and Ellwood et al. [16] we assume that wind is not strong enough to cause the coarse fraction to saltate and that saltating fine grains drive the coarse grains into reptation. It is a two scales model where spatial variations of the saltation flux dominates at large scale (order of meters) and for long times, while spatial variations of the reptation flux dominates at small scale (order of centimeters) and for shorter time scale. The megaripples wavelength is approximately 4 times the mean saltation length. The analysis shown here is linear and as the ripples grow, nonlinear effects become important. Thus, the predicted wavelengths are correct only for short times and the final wavelength has to be found from numerical solutions or from nonlinear analysis. The proposed model can explain the formation of huge megaripples seen on Mars, where the thin Martian atmosphere greatly increased the saltation length [9]. Provided real values for the various parameters, the model can be used to predict megaripple wavelength and the minimum percent of coarse grains at the surface needed for megaripple formation. By including the saltation flux of the coarse grains in the Exner equation, the model can account for situations where the wind is very strong. We intend to investigate these effects in a future study.

5. Uncited references

[15,19]

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