EFFECT OF MASS ON DOWNHILL CYCLING: DOES THE BIKER'S WEIGHT HELP?

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Abstract

In high-school teaching of mechanics, we deal, among other things, with the nature of static and kinetic friction, forces that are proportional to the normal force. Under the influence of frictional forces, a body moves down a rough sloped decline at a fixed rate of acceleration that is independent of its mass. This situation does not apply to cases where the frictional force is dependent upon velocity, such as bodies which are moving through a streaming fluid (such as raindrops falling to the ground). In this case the body moves with a continuously decreasing acceleration, eventually reaching a terminal velocity when the frictional and gravitational forces balance out. This velocity constraint is determined by the dependence of the frictional force on velocity and geometric parameters that determine the strength of the frictional force. We show here that a similar situation takes place when bicycles descend an incline with a fixed slope. We also investigated the dependence of the velocity constraint with mass, using bicycles equipped with sophisticated sensors that metamorphose them into data-processing laboratories.

Keywords

Bicycles, Terminal velocity. Air drag, Rolling Friction, Polar S725X, Physics teaching

1. INTRODUCTION

The system of a bicycle and rider moving in a straight line is affected by a number of forces. It should be noted that this system cannot be represented as a simple point-like body and there is great importance to understand the background of how these forces operate. By pushing the pedals the cyclist created the rotational momentum that turns the front chainring which via the chain turns the rear chainring, attached to the rear wheel itself. The rear wheel turns and creates the frictional force on the ground which is directed backwards. According to Newton’s Third Law of Motion, the ground exerts an equal but opposite force on the bicycle, which is the force that actually propels the bike. Fig. 1 illustrates the forces operating on a bicycle moving up an incline in a straight path.

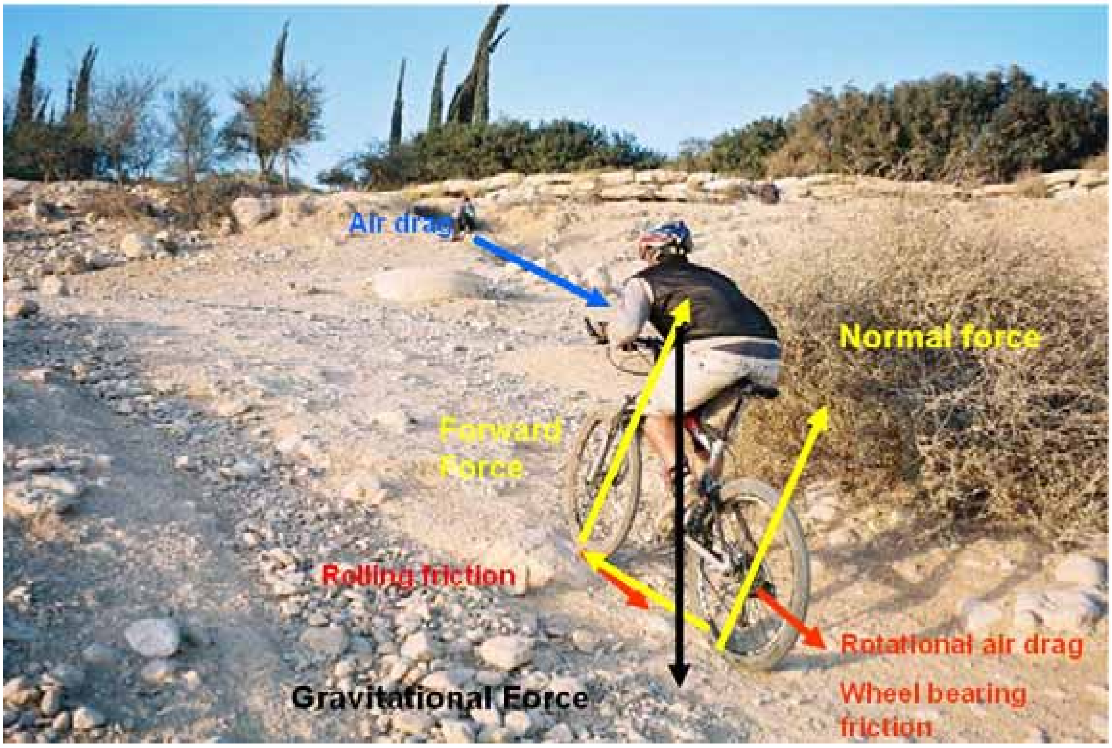

Fig. 1 The forces operating on a bicycle moving up an incline in a straight line. There are two main friction forces; the first is the rolling friction which dominates at low speeds while the air drag is dominates at high speeds.

The equation of motion for the moving bike can be written as follows [1, 2] :

1. $ma = F_p - (F_A + F_S + F_R),$

where F_p is the force pushing the bike, F_A is the dragging force of the air, F_s is the opposing force produced by the incline, F_R the frictional force of rolling caused by the deformation of the tire and the ground. m is the total mass of the rider and bicycle, and a is the acceleration of the rider and bicycle. On a relatively smooth road, the important forces are the translational air drag and rotational friction. The drag or frictional force of the air is caused by the pushing aside of air with the movement of the bike and the formation of a higher pressure pocket of air in front of the rider and a lower pressure pocket behind, and also from the friction of the moving air along the front surface of bike and rider. It is important to note that this force is dependent on **the square of the speed of the bike**. Over 15 km/hr, this force dominates [3]. Air drag can be expressed more accurately as follows:

2. $F_A = K_d (v + v_w)^2,$

where v is the bicycle speed with respect to the ground and v_w is the air speed (with $v_w > 0$ for an opposing wind). The coefficient K_d is dependent on the cross-section of the rider and bicycle perpendicular to the direction of movement $-A$, the variety of bicycle frame design, and the density of the air. This coefficient can be expressed as [4]

3. $K_d = \frac{1}{2} c_d A \rho.$

The dimensionless drag coefficient c_d is nearly constant and as a first approximation does not depend on velocity. The drag coefficient is primarily the result of eddy currents that develop in the air posterior to the moving bicycle. For example, for a sport bike, $c_d = 1$ when the rider is sitting vertically but is reduced to 0.9 when the rider is bent over. The typical cross section of the rider, when sitting vertically is *A* = 0.4 m^2. *ρ* is the air density, which at sea level is 1.2 kg/m^3 . Using known factors, we can calculate the air drag for a sport bike (with a vertical rider) moving at 36 km/h (10 m/sec) under windless conditions as

4. $F_A = 0.5 \times 1.0 \times 0.4 \times 1.2 \times 10^2 = 24$ N.

The frictional forces of the wheels depend on the normal forces acting on each wheel, and with the coefficient of rolling friction c_r depending on air pressure, area and cross section of the tire, the wheel diameter, and the ground roughness itself. We can, to a good approximation, consider the two normal forces as acting as a single force N and similarly represent the frictional forces of rolling operating on both wheels as a single force, as follows:

5. $F_R = c_r N.$

For racing bikes, the typical coefficient of rolling friction $c_r = 0.003$. For example, for a total mass of 85 kg, the rotational friction on flat ground can be written as:

6. $F_R = 0.003 \times 85 \times 9.8 = 2.50$ N.

It is important to note that the rotational friction does not depend on speed (in a significant way), while the air drag is dependent on the square power of velocity. This last fact has a fundamental effect on the speed obtainable by a bike. In order to reach maximal velocities, the air drag must be reduced by improving the aerodynamic profile of the bicycle (decreasing c_d). This is accomplished by building a special envelope around the bike in which the bicyclist is always in a sitting, recumbent position. These bicycles are known as human powered vehicles (HPV) and annual competitions take place to break the previous record speed traveling on level ground or the record distance that can be covered during an hour. Competition bikers reduce air drag by riding in a close-packed queue, a technique known as drafting. In this case, the bikers towards the back save about 20% of the energy input required to maintain the same speed, when biking alone. (See Fig. 2.) Since air drag is proportional to v^2, the mechanical power required to overcome it increases as v^3. For this reason, great efforts have gone into approaches to decrease air drag.

For bicycles going down a slope with an angle of α, the equation of motion can be written as [5]:

7. $ma = mg(\sin\alpha - c_r \cos\alpha) - K_d v^2.$

The terminal speed v_t will be attained when the bicycle's acceleration will become zero i.e. when $mg(\sin\alpha - c_r \cos\alpha) = K_d v^2$ and we come up with the following expression for v_t:

8. $v_t = \sqrt{\frac{mg(\sin\alpha - c_r \cos\alpha)}{K_d}}$.

The expression of speed as a function of time is given as:

9. $v(t) = \gamma \tanh(\psi + t/\sigma),$

where $\gamma = \sqrt{F_0 / K_d}$, $F_0 = mg(\sin\alpha - c_r \cos\alpha)$ and $\sigma = m/\sqrt{F_0 K_d}$. The constant Ψ is determined from the initial velocity according to the relationship: $v(0) = \gamma \tanh\psi$.

Fig. 2: The phenomenon of drafting in the flight of cranes, bicycling, and motor racing. The goal here is to move in close formation behind the moving body, a position in which air resistance is reduced, resulting in energy conservation. In the case of bicycling, the rider taking advantage of drafting can save up to 30% of the power exerted when traveling alone at the same speed [6].

The equation of motion of a bicycle going uphill on an inclined surface is given by:

10. $ma = F_p - mg(\sin\alpha + c_r \cos\alpha) - K_d v^2$.

Assuming that the ascending speed is small (the hill is steep), air drag may be ignored. In addition, assuming that the rider pedals at a fixed level of power, his speed will decrease with an increase in total mass. For moderate inclines, where air drag cannot be neglected, the added time for uphill biking will still be greater than the time made up in the downhill segment, because the time spent peddling by the rider during uphill is greater than the time he coasts downhill. Therefore in cycling competitions, such as the Tour de France, there is a particularly important advantage for low-weight riders in the hilly regions. However, despite the advantage for higher-weight bikers[4] in downhill riding, in hilly, up and down, regions there is a clear advantage for lower-weight bikers due to the additional time required to pedal up the inclines [7].

In order to illustrate the advantage of heavy bikers down a decline[5], we can for simplicity assume the rider's body as a uniform cylinder (a "cylindrical biker") which has a density of ρ_r , a height of h and a diameter D. Then the cylindrical biker's mass will be $\pi D^2 h\rho_r / 4$ and his cross section area A will be Dh. We can then express the cross-section area in terms of mass as follows:

11. $$A = \frac{4m}{\pi D \rho_r}.$$

Utilizing this result and Equation (3) and substitution into the equation we obtained for the terminal (Equation 8) velocity, we get

12. $$v_t = \sqrt{\frac{\pi g \rho_r (\sin\alpha - c_r \cos\alpha) D}{2\rho}}.$$

At first glance, it appears that the terminal velocity does not depend on mass. However, taking a closer look, we see that for the cylindrical rider, this velocity is proportional to $D^{1/2}$, that is, the square root of the body width of the biker. Assuming that all riders have the same height and density, the difference in mass must be expressed in the body width. Namely more bulky bikers will have higher values of D and their terminal velocity will be higher than their thinner counterparts.

2. DESCRIPTION OF THE EXPERIMENTAL APPROCH

In order to study the influence of mass on the terminal velocity of bicycles moving down a decline, we used mountain bikes fitted with sensors and a data gathering Polar 725s (Sports Heart-Rate Analyzer/Altimeter/Computer) containing the power meter option. The device illustrated in Fig. 3 can measure and store data at the maximum resolution of every 5 seconds. It can measure bike speed, cadence, altitude above sea level, and the power exerted by the rider by multiplying the chain tension by its velocity. It also measures the biker's heart rate via a chest strap. All these sensors send data back to the unit installed on the bicycle. This device is one of the four power measuring devices that are commercially available and was chosen due to its high accuracy and relatively low cost. It this experiment we did not utilize the power meter option because the power exerted by coasting downhill is zero, but rather took advantage of its ability to record the instant velocity each 5 seconds. We changed the "rider's mass" by adding weights from exercise rooms to the backpack worn by the biker. We got to a maximum of 40 kg of added weight. We found a road with a fairly constant angle of inclination and which was lightly traveled by automobiles. It was the road going from the Ziporim Intersection to Ramat Matred, about 8 km south of Midreshet Sede Boqer located at the southern part of Israel. We carried out the experiment before noon in order to minimize the influence of natural winds. In general, this area experiences western winds of about 6 m/sec in the afternoon hours. According to Equation 8 the terminal velocity increases as the square root of the total mass of the rigged bicycle and the biker. We attempted here to determine whether this relationship is realistic in the field.

Fig. 3: Schematic diagram of the Polar S725X used to measure power exerted by the biker. The various factors are stored in a display unit attached to the bicycle and may be downloaded via an infrared device to a laptop computer. (Source: http://www.polarusa.com/Products/consumer/s725x.asp).

3. RESULTS

Fig. 5 presents a sample curve showing how velocity changes with time for a riding mass of 106.4 kg. From the curve one sees how the increase of speed of the bike gets smaller and smaller, i.e. its acceleration decreases, until it reaches its terminal velocity of about 42 km/hr and zero acceleration. The displacement as a function of time can be obtained by time integration of the speed.

Fig. 6 shows the displacement as a function of time during biking down the decline. One can see that in the latter portion of the graph the curve becomes linear, showing that the bicycle has attained a fairly constant speed.

Fig. 4: Carrying out the experiment on the declining road between the Ramon Base to and the Ziporim Intersection, south of Midreshet Ben Gurion. The

decline in the segment we chose is approximately constant and traffic there is very light. The best time to carry out this experiment is early in the morning in order to reduce wind effects on the experiment. The experimental distance should be at least half a kilometer.

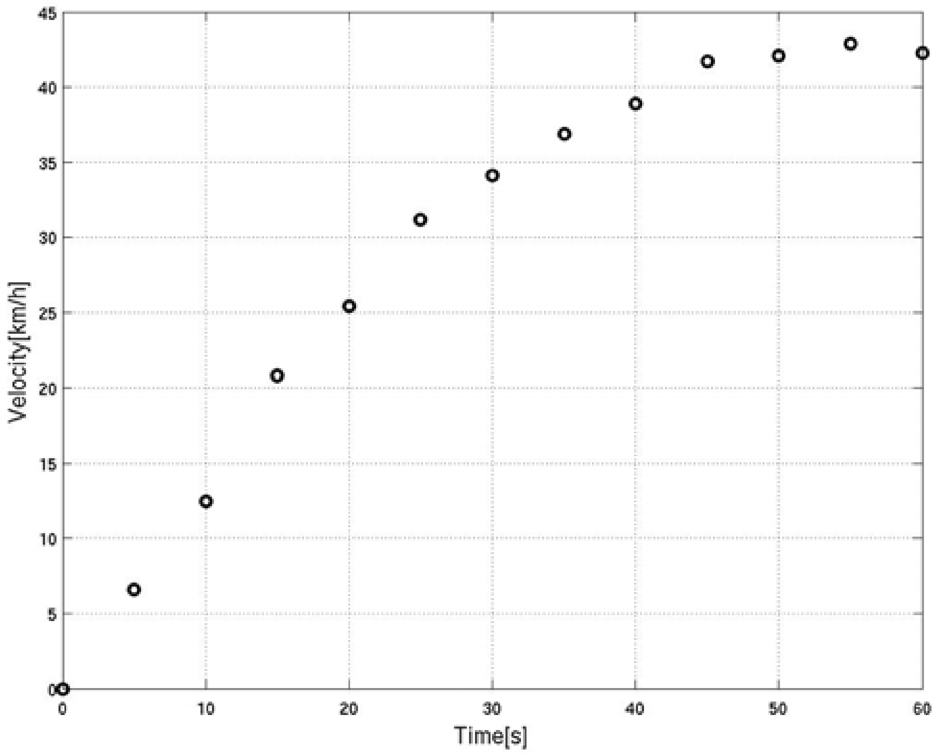

Fig. 5: The results of measuring bicycle velocity on a decline in units of km/hr as a function of time (seconds). For this coasting experiment the bicycle moves under a continuously decreasing acceleration, due to air drag, which increases with speed. After about 50 second the net forces operating on the bike cancel out, with the bicycle, in this case, continuing to move at a fixed velocity of about 42 km/hr. According to the experiment described in the curve the total rider-bike mass was 106.4 kg.

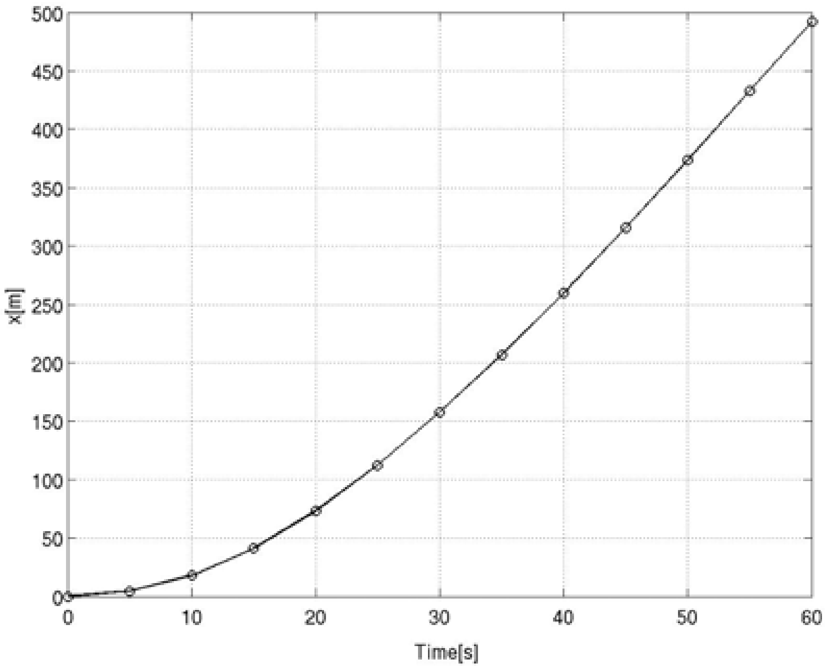

Fig. 6: The displacement as a function of time for the experiment described in Fig. 5. The displacement was calculated as the normal by integrating the velocity as measured by the Polar system.

In the experiment, we increased the mass in intervals of 5 kg, and instructed the biker to preserve his riding position in order to preserve the same drag coefficient for each ride. The results are shown in Table 1:

Final velocity (km/h)	**Total mass of bike and rider (kg)**
40.0	96.7
43.3	101.5
41.7	106.4
46.2	111.2
46.1	116.1
45.0	120.9
44.8	125.8
46.4	130.6
52.6	135.5
47.4	140.3

Table 1: The table reports the values of final velocity for coasting downhill for different masses. The experiment lasted about two hours, and from time to time wind blasts contributed to the final velocity, producing a scattering of the data.

In Fig.7, we plot $\ln(v_t)$ as a function of $\ln(n)$. According to Equation 7, the equation of the straight line is:

13. $\ln v_t = \frac{1}{2}\ln m + \frac{1}{2}\left[\frac{g(\sin\alpha - c_r\cos\alpha}{K_d}\right]$.

The slope of the straight line is 0.49, close to the theoretical value 0.5. The spread of points about the line is primarily due to velocity shifts produced by individual blasts of wind that occurred during the experiment. Naturally outdoor experiments do not allow optimal conditions with fixed or zero wind exposures. These blasts can be as high as 5 m/sec and are characteristic of turbulent atmospheric air flow [8]. We carried out our experiments in the afternoon hours in order to minimize the effects of wind. However, it would have been preferable to perform them early in the morning before the air begins to be heated by the sun. Because air drag is proportional to the square power of velocity of the bicycle with respect to the wind, the wind can significantly influence the terminal velocity of the bike. Other factors influencing the point spread include the small changes in the riding position of the biker, which are extremely difficult to control. In our analysis, we also ignored the increase of the coefficient of rolling friction produced by increasing rider weight. However, at high speeds, rolling friction is negligible with respect to wind drag [3].

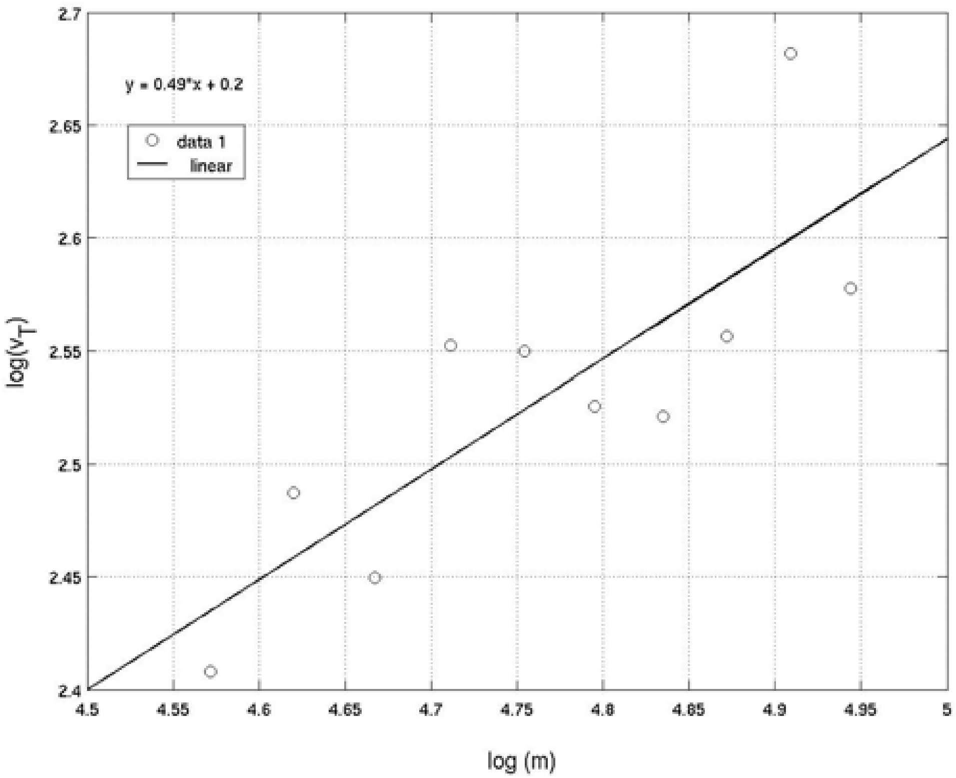

Fig. 7: Plot of $\ln(v_t)$ as a function of $\ln m$ for downhill experiment. The linear approximation of the slope is 0.49, extremely close to the theoretical slope of 0.5. The equation for the linear approximation is $\ln(v_t) = 0.49\ln m + 0.2$.

4. SUMMURY

Bicycles equipped with a configuration of Polar 725c sensors constitutes a virtual, automated laboratory that enables the biker to actively contribute to experimental studies. It therefore provides an unusual tool for science teaching. Using this bicycle various experimental tasks can be carried out as enrichment projects in the teaching of mechanics in 11th grade high-school classes. The chapters dealing with work and energy can be illustrated, as well as the measurement of power (which we have not detailed in this article). The student can carry out measurements with his body, thereby increasing his interest and motivation in the study of physics. In this report we have shown how mass influences the terminal velocity of bikes coasting down a hill. In physics classes, we normally teach systems in which the force of friction is proportional to the normal force and speed is independent of mass. But for objects moving through air, the frictional force produced (air drag) is dominant and depends on the square power of the object's speed. In this case more massive bodies fall faster than lighter bodies. We have shown that the terminal velocity of bicycles, to a good approximation, increases proportionally to the square root of its mass. Therefore, the heavier bikers will be able to reach a higher maximum velocity than their lighter counterparts. However, in uphill pedaling the heavier biker is at a clear disadvantage. For this reason the winners of contests such as the Tour de France are the strongest uphill bikers.

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