The freewheeling cyclist

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This article is concerned with the so-called 'freewheeling method' for direct and accurate determination of rolling friction and the air drag factor of rolling vehicles (e.g. a bicycle). The method is of practical importance in the sport of cycling, and can be used to determine the power output of an individual cyclist.

PART I: PRINCIPLES, THEORY AND INSTRUMENTATION

The previous article published in this journal [1] by one of the authors was mainly concerned with the more efficient use of mechanical power delivered by a cyclist. As stated in the conclusions, practical tests performed on bicycles should include determination of the frictional forces acting on both the bicycle and the cyclist. One approach to determine rolling resistance (F_r) and air drag factor (K_d) is that of freewheeling on a level road; this method is based on the measurement of a decrease in speed as a function of time. No forces other than rolling friction and air drag are assumed to act on a cyclist; such an experiment (described by Minnaert [2] in 1942) is easily carried out by any cyclist. In his book Minnaert explained how such a decrease in a bicycle's speed can be manually recorded (by an assistant to the cyclist). Minnaert demonstrated how values for F_r and K_d can be simply deduced from such measurements by plotting the (negative) acceleration of the freewheeling bicycle versus speed squared. As deceleration is proportional to the squared speed, this plot is a straight line (see equation (1)).

The introduction of digital cycling computers in the 1980s was the inspiration for one of the authors to carry out a new experiment. In their 1991 experiment Hennekam *et al* [3] used a dictaphone to register (recording by auditive signal) the readings of decreasing speed from a digital cycling computer attached to a test bicycle. Compared with Minnaert's method for obtaining F_r and K_d the accuracy achieved in determining these parameters in this experiment was substantially improved due to the use of a microcomputer program (MATLAB [4]).

The MATLAB calculations produced satisfactory results when compared with values found in the literature, starting from mean values of recorded data *in repeated measurements*†. However, when using a dictaphone the outcome of *individual measurements* exhibits a rather large scatter of recorded data points. This is also expected, since the reaction time of the cyclist reduces both the accuracy and the number of data points.

The accuracy of a single measurement can be considerably improved by recording the speed electronically. The aim of our recent study was to achieve such additional enhancement of the 'freewheeling method', allowing one to determine the rolling friction and air drag factor in a quick and reproducible way. As in the previous experiment, the MATLAB program was used to fit experimentally obtained data directly to the mathematical solution of the equation of motion describing freewheeling on a level road†† (see the Theory section below).

The last section of Part I explains how the new experiment was set up, giving some details about the apparatus used. The experimental procedure and results are presented in Part II of this article, which also contains an Appendix with more detailed information on the electronic device ('signal converter' used to carry out these new measurements.

Figure 1. *Bicycle and a cyclist during the test.*

Theory

Assuming

$$F_w = F_r + F_d = F_r + K_d v^2 \qquad (1)$$

†For example, repeating freewheeling with the bicycle five to ten times *in both directions*: taking mean values of recorded data reduces possible effects of wind speed and inclination of the road surface, as suggested by Minnaert.

††This method is based on the nonlinear regression method of Nelder-Mead ('Simplex').

for a total frictional force F_w acting on the freewheeling bicycle, one can solve the equation of motion for a freewheeling regime [3]. As seen in equation (1), the model assumes a constant [5] value for rolling resistance F_r and a speed-dependent [6] proportionality factor; this factor F_d is the air drag factor mentioned above.

If F_r and F_d are the only forces acting on the bicycle, the equation of motion is

$$m \frac{dv}{dt} = - F_r - K_d v^2 \qquad (2)$$

In equation (2) the minus signs indicate that the frictional force F_w is opposite to the direction of motion; m is the total mass of the bicycle and cyclist.

The solution $v(t)$ of this differential equation (2) (discussed in the previous article [3]),

$$v(t) = \beta \tan (\phi - t/\tau) \qquad (3)$$

indicates a decrease of speed v with time for the freewheeling cyclist moving in still air and on a level road. The scalar parameters β and τ defined by

$$\beta = \sqrt{F_r / K_d} \quad \text{and} \quad \tau = m / \sqrt{F_r K_d}$$

are related to the physical quantities F_r and K_d through

$$F_r = m\,\beta/\tau \quad \text{and} \quad K_d = m/\beta\tau. \qquad (4)$$

The phase factor ϕ in equation (3) is determined from the requirement imposed on the initial speed, i.e. $v(0) = v_0 = \beta \tan \phi$.

Experiment

As mentioned above, the MATLAB computer program was used here to compare the mathematical solution (3) (describing freewheeling motion on a level road) directly with experimentally obtained data.

Hence, if the speed–time profile can be precisely recorded (during freewheeling on a level road), MATLAB allows for direct and accurate determination of the desired parameters (F_r and K_d). Such accurate measurements were achieved here by the electronic recording of (electrical) pulses, generated in an inductive sensor mounted on the front fork of the bicycle. Elimination of errors due to slow reaction time (of the test rider using a dictaphone), and an increased number of (v–t) data points led to more reliable values for F_r and K_d (compared with those in the previous experiment). The electrical pulses were induced by the magnetic field formed by 28 small magnets (arranged symmetrically on a central ring), attached to the spokes of the bicycle's wheel; the sensor and magnet ring used in this experiment were those of a conventional cycling computer (Axa cyclotronic [7]). Pulses appearing as an alternating signal at the sensor (28 pulses per wheel revolution) were detected by a portable audio-recorder. With a wheel 70 cm in diameter, pulse frequencies were between 15 and 200 Hz, corresponding to a velocity range extending from 5 to 55 km h^{-1} (frequency is proportional to the rotational speed of the wheel and hence to the speed of the bicycle itself).

The battery-operated mono cassette-recorder (Sony TCM-919) used in this experiment registered only frequencies in the range 150–6300 Hz. In order to allow reliable registration with such a simple audio-recorder, the signal generated by the sensor had to be transformed first. Therefore, a signal converter (SC) was developed by our departmental electronic workshop; more details on the wiring of the SC and its operational characteristics are given in Part II of this article (Appendix).

Following completion of the experiment, the frequencies recorded on a tape were stored in a data acquisition unit (Hewlett Packard HP 3421A) in the laboratory; this device counts the number of pulses every second. Because the accuracy in registering the speed depends on sampling time, the use of a datalogger instead of the cycling computer is recommended (reducing sampling time to one second or less)[†].

A battery-operated portable data acquisition unit could be used instead of the Signal Converter/HP-3421A datalogger combination; however, this is not very practical (weight!).

Finally, for any pair of (v, t) data points found, MATLAB calculates the corresponding, most accurate values for F_r and K_d.

[†]Most cycling computers calculate average speeds over quite a long period of time (several seconds); the AXA cyclotronic has an (estimated) sampling time of 2–3 seconds.

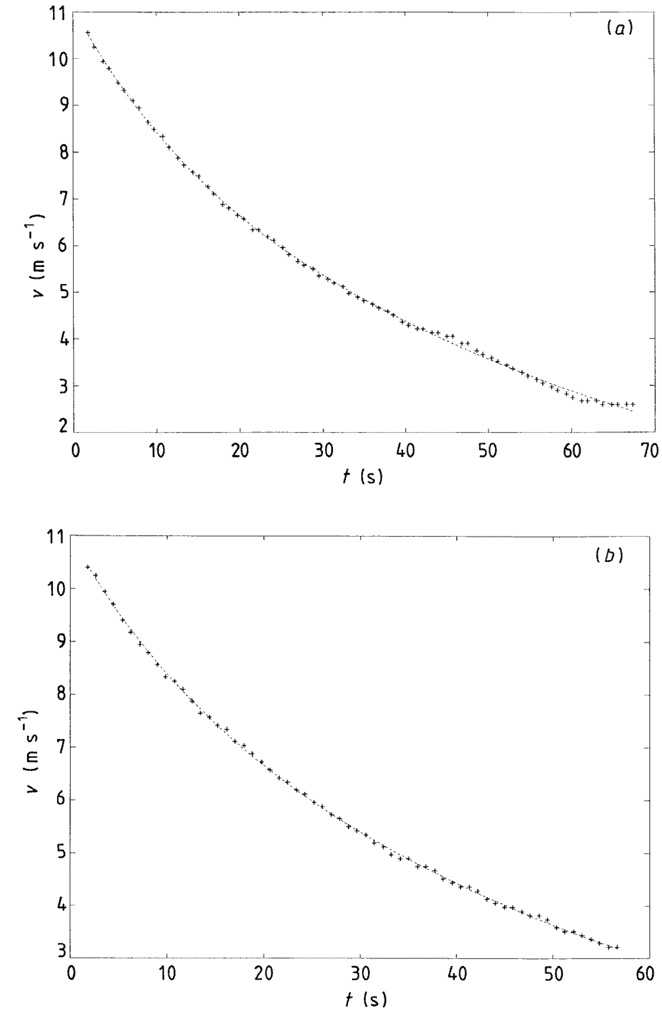

Figure 2. *Graphical representation of the computed best fit (single measurements) in the case of (a) strong oscillations and (b) small oscillations.*

PART II: EXPERIMENTAL PROCEDURE AND RESULTS

As mentioned in Part I of this article, this work describes a substantial improvement of the 'freewheeling method'. Recording a decrease in speed electronically provides results that are more reliable than those from an earlier experiment, where a dictaphone was used.

The freewheeling experiment

The new experiment was carried out in the open air and under wind-free conditions. In order to check reproducibility, all measurements were performed several times and *in both directions*.

When comparing measured data obtained from repeated measurements *in only one direction*, some degree of scatter is still observed. The smoothness of the road surface proved an important factor, as evidenced by less accurate results obtained on a bumpy road. The effect of a small track inclination could be accounted for, by averaging values for F_r *in both directions*. This is easy to understand: if the inclination angle ϕ is assumed constant, the constant force in the differential equation (2) contains an additional term $F_g \sin\phi$, corresponding to a component of the gravitational force F_g parallel to the road surface. This leads to two different values for the constant force in equation (2): when riding up or downhill, the components of the gravitational force are in different directions and hence cancel each other (exactly). The mean value of these two constant forces equals F_r.

As an example, for an inclination angle ϕ of 0.001 rad ($\approx 0.05°$) $F_g \sin\phi \approx 80 \times 10 \times 0.001 = 0.8$ [N]; hence this effect cannot be neglected.

Discussion of experimental results

In graphs showing results obtained for a single measurement (in one direction) the computed, best fit suggests a susceptibility of the bicycle towards small oscillatory movement with respect to the theoretical line (see figure 2(*a*)).

This phenomenon can probably be ascribed to the effort of pulling on the handlebars whenever the bicycle's equilibrium is being disturbed (in particular at lower speeds). Inspection of these graphs shows that a small oscillation appears at speeds below 4 m s^{-1} (approx. 15 km h^{-1}); consequently, great care should be exercised to avoid excessive oscillatory movements of the bicycle at low speeds. This effect can be compensated for by combining measurements *in the same direction* (i.e. taking mean values; see table 1).

All values for K_d found in different experiments are (within their accuracy) in good mutual agreement. Using some 30 to 40 pairs of data points for *v* and *t*, the accuracy *of a single measurement*, as calculated by computer, is 5–6 %. Use of more data points (60 – 80) will enhance this accuracy to 2–3 % (table 2).

Determination of F_r *from a single measurement* proved less accurate, as could be expected from earlier considerations (i.e. bumps in the road surface and problems associated with balance at low speeds), and therefore only values of K_d are shown in table 2.

The experiment was carried out on a Giant Peloton Lite racing bicycle. By combining measurements of this experiment (tables 1 and 2) the most accurate numerical values for F_r and K_d were

$$F_r = 2.30 \pm 0.08 \text{ N}$$
(inflation pressure approx. 7–8 bar)

Table 1. Mean values of F_r and K_d obtained in a series of measurements *in the same direction* (back/forth).

	F_r (N)	ΔF_r	K_d (kg m^{-1})	ΔK_d
Back				
	2.05	0.09	0.203	0.009
	2.21	0.07	0.201	0.007
	2.24	0.10	0.204	0.009
	2.04	0.07	0.203	0.007
	2.09	0.07	0.205	0.007
	2.21	0.07	0.202	0.006
Forth				
	2.49	0.08	0.204	0.007
	2.54	0.09	0.203	0.008
	2.47	0.09	0.212	0.008
	2.37	0.10	0.209	0.009
	2.44	0.08	0.202	0.007
	2.52	0.09	0.206	0.008

$$K_d = 0.205 \pm 0.007 \text{ kg m}^{-1}$$
(racing position/racing gear).

These calculated values agree well with those found in the literature [1], and are also comparable to the results of the previous experiment [3].

An alternative method, not tested yet†, relies on a determination of speed calculated from time intervals elapsed between consecutive pulses (electronic timer-counter). Such an approach increases the number of data points considerably. Another way to achieve this (one does not need the datalogger or a timer-counter now) would be to use a video camera to register speeds at shorter time intervals. The latter method is also interesting from an educational point of view.

Conclusions

The method described here enables a quick and reliable determination of the rolling resistance F_r and air drag factor K_d. The outcome of this study is interesting from the physical point of view;

†Such a timer-counter was not yet available for our experiments.

furthermore, the results are useful (as a calibration method) in applied sciences such as ergonometry and sport medicine. In addition, the obtained numerical values for F_r and K_d can be used to study the influence of the rider's position (racing or reclining bicycle) and the effects of a windscreen as well as a streamlined bicycle enclosure on the human effort required in the sport of cycling.

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Appendix. The signal converter

The signal converter (SC) transforms a low-frequency sine wave signal (generated in the sensor) into a block pulse of the same frequency; this in turn drives a 5 kHz carrier wave. The 5 kHz signal is recorded on a tape through the microphone plug of the recorder. After the experiment, the tape can be restarted. The recorded 5 kHz signal is connected to the earphones plug and transformed into a block wave (of the same frequency as the original sensor signal) by the SC. This retransformation was achieved by means of a so-called retriggerable monostable multivibrator ('one shot'; see figures 3 and 4).

The wiring of the SC. When *recording*, the signal representing the speed will be transmitted through P1 (figure 5), and appears as a sine wave voltage (frequencies of 15–200 Hz; peak-to-peak amplitudes between 1 and 5 V).

The negative part of the signal is cut off by a Zener diode, while the positive part is limited at 5.6 V. The signal is then amplified (via T1 and T2) and limited (by the DC voltage of the power supply) to produce a square block wave; T2 also drives an LED for control purposes. The wiring containing IC1a and IC1b produce a 5 kHz square block voltage; the 5 kHz from this generator is switched on/off (in IC1c) by the sensor signal block pulse; IC1d acts as an inverter. The output signal from IC1d is then filtered

Table 2. Values of K_d with 60–80 data points obtained *by a single measurement.*

Back		Forth	
K_d (kg m^{-1})	ΔK_d	K_d (kg m^{-1})	ΔK_d
0.208	0.006	0.213	0.010
0.209	0.005	0.212	0.011
0.207	0.006	0.205	0.007
0.199	0.005	0.201	0.005
0.202	0.005	0.207	0.005
0.203	0.006	0.200	0.007
0.201	0.005	0.213	0.008

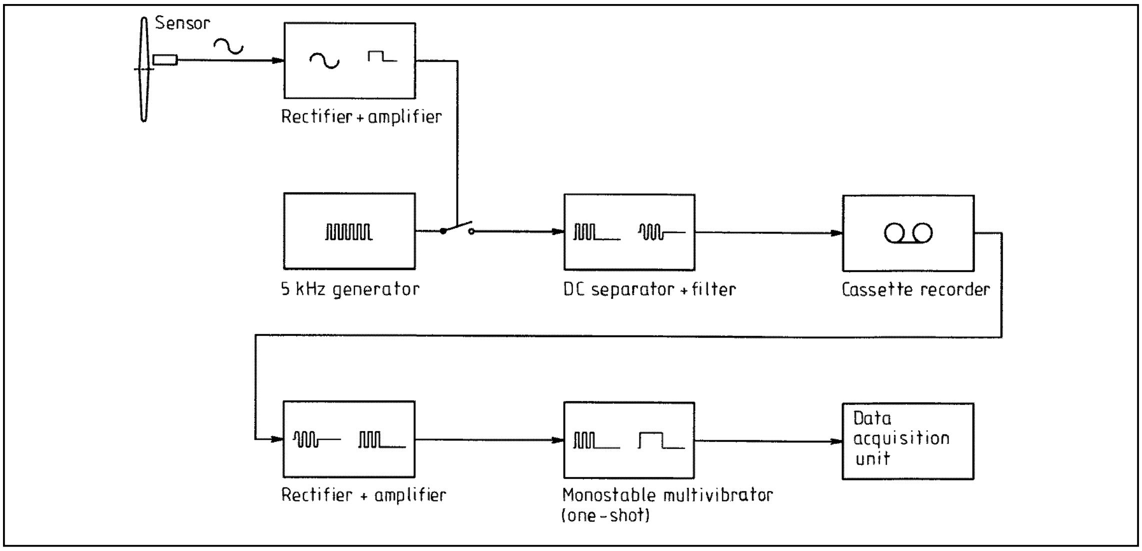

Figure 3. *Block diagram of the signal converter (SC).*

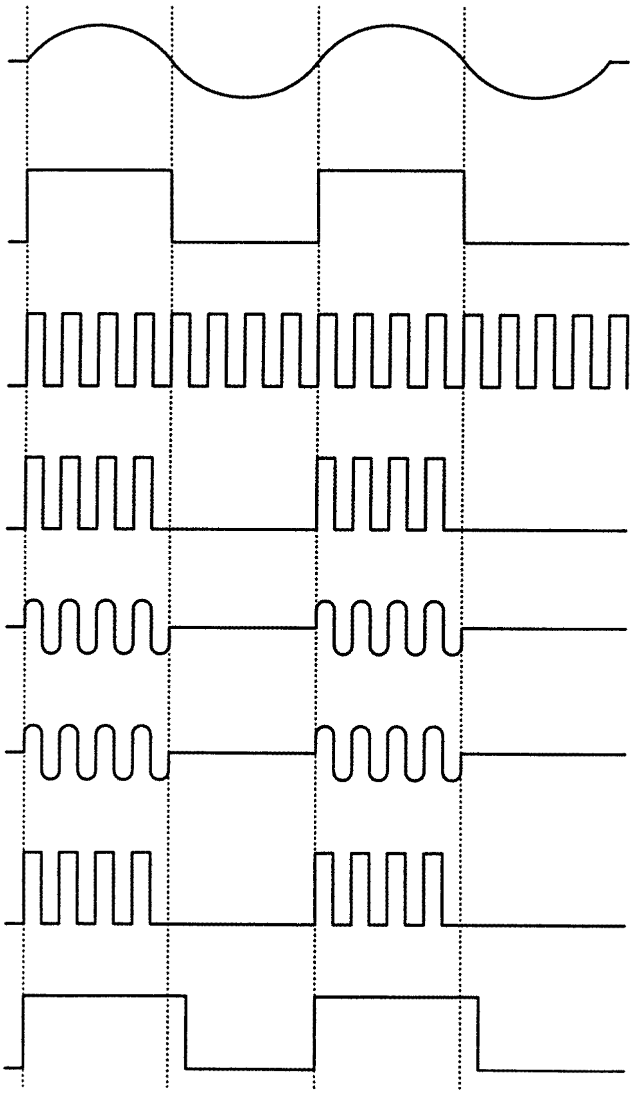

Figure 4. *Transformation of the original sine wave signal.*

(by an *RC* circuit), reduced and finally AC-coupled to the microphone input of the cassette recorder.

When *replaying* the tape, the earphone signal is supplied through P2. A high-pass filter allows only high frequencies to pass and a pinch/trap diode cuts off the negative parts of the signal; the remaining signal is then amplified (via T3) and restricted by the voltage of the power supply; another LED is driven to control the signal. The IC2 is connected as a retriggerable one-shot having a time period of 660 μs; the incoming signal (5 kHz) has a corresponding time period of 200 μs. The output of IC2 will remain high for about 660 μs when a falling edge passes the input; if another falling edge arrives within this period, the output signal remains high. In such a way, the recorded 5 kHz signal is retransformed into a square block wave at the same frequency as the original signal generated at the sensor.

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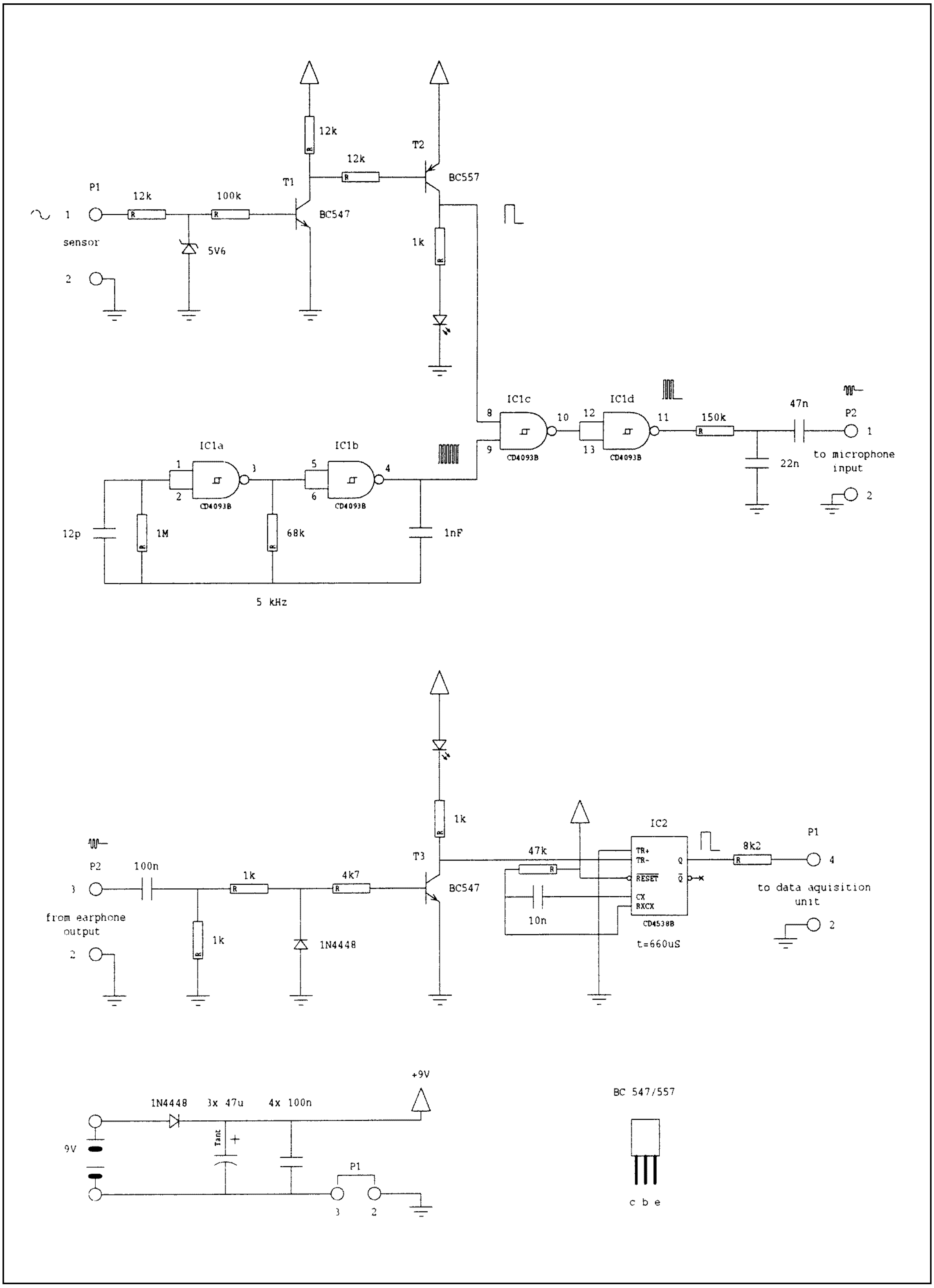

Figure 5. *The wiring of the SC.*

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